# Resit Exam Introduction to Logic (CS & MA)

Thursday 26 November 2015, 18:30 - 21:30 h.

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- Write with a *blue or black pen* (so no pencil, no red pen).
- Write your *name and student number* on all pages of your work.
- Only hand in your definite answers. You can take the exam questions and any drafts home.
- With the regular exercises, you can earn 100 points. With the bonus exercise, you can earn 10 extra points.
  The resit exam grade is the number of points you earned divided by 10, with a maximum of 10.
  The grade for this resit exam is also your final grade. The results of the homework assignments, the midterm exam and the regular exam do not count.

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1. (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.
   (a) Miko will come, provided that it neither rains nor snows.
   (b) Sam likes coffee, but he only drinks it if he is not fully awake.
2. (15 points) Translate the following sentences to *first-order logic*. The domain of discourse is the set of students. Provide a single translation key for both sentences (5 points).
   (a) Nobody is his/her own mentor's mentor.
   (b) Everyone whose mentor has Amir as mentor, knows someone who is not the mentor of any student.
3. (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers.
   (a) Is $(A \leftrightarrow B) \leftrightarrow ((C \leftrightarrow \neg A) \leftrightarrow (B \leftrightarrow C))$ satisfiable?
   (b) Are the following two sentences tautologically equivalent?
      i. $(\neg A \rightarrow B) \wedge C$
      ii. $\neg(A \wedge C) \rightarrow B$

4. (20 points) Give formal proofs of the following inferences. Do not forget the justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

(a) $A \to (B \vee C)$

$\neg B \to (\neg C \to \neg A)$

(b) $(A \leftrightarrow B) \leftrightarrow C$

$A \to (B \to C)$

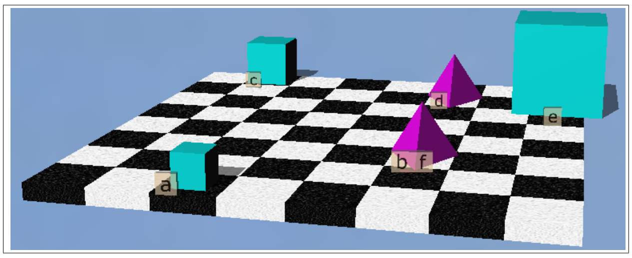
(c) $\exists x(a = x \wedge x = b)$

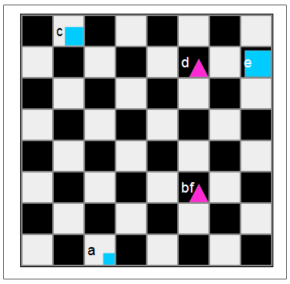
$\forall x(x = a \to x = b)$

(d) $\neg\exists x(P(x) \wedge \neg Q(x))$

$\forall x(P(x) \to Q(x))$

5. (15 points)




In the world displayed above, a is small, e is large, and the objects named c, b, d, f are medium.

(a) In the world displayed above, no two cubes are in the same row. Express this with one sentence in the language of Tarski's World. The sentence should be true in all worlds with no two cubes in the same row, and false in all worlds with two cubes in the same row.

(b) Indicate of the sentence below, whether it is true or false in the world displayed above. Explain your answer.

$$\forall x(\mathrm{Tet}(x) \to \exists y(\mathrm{SameCol}(x, y) \wedge \exists z(\mathrm{SameRow}(y, z) \wedge \mathrm{Cube}(z))))$$

(c) Indicate how the sentence below can be made **false** by removing one object from the world displayed above. Explain your answer.

$$\forall x(\forall y(\mathrm{SameCol}(x, y) \to x = y) \to \mathrm{Cube}(x))$$

6. (15 points) Let a model $\mathfrak{M}$ with domain $D = \mathfrak{M}(\forall) = \{0, 1, 2\}$ be given such that

$$\mathfrak{M}(a) = 0 \quad \mathfrak{M}(b) = 1 \quad \mathfrak{M}(c) = 2$$
$$\mathfrak{M}(P) = \{0, 2\} \quad \mathfrak{M}(Q) = \{1\}$$
$$\mathfrak{M}(R) = \{\langle 0, 0\rangle, \langle 2, 0\rangle, \langle 2, 1\rangle, \langle 2, 2\rangle\}$$

Let $h$ be an assignment such that $h(x) = 2$, $h(y) = 0$, and $h(z) = 2$. Evaluate the following statements. Follow the definition of satisfaction (truth definition) step by step.

(a) $\mathfrak{M} \models R(x, z) \wedge (P(a) \vee Q(b))[h]$

(b) $\mathfrak{M} \models \forall x(R(c, x) \rightarrow Q(x))[h]$

(c) $\mathfrak{M} \models \exists x \forall y R(x, y)[h]$

7. (15 points)

(a) Provide a disjunctive normal form (DNF) of the sentence $\neg(A \leftrightarrow (B \vee \neg C))$. Show all intermediate steps.

(b) Provide a Skolem normal form of the sentence $\neg(\forall x A(x) \rightarrow \forall x \exists y \forall z B(x, y, z))$. Show all intermediate steps.

(c) Check the satisfiability of the Horn sentence. Use the Horn algorithm and indicate the order in which you assign truth values to the atomic sentences.

$$((A \wedge B) \rightarrow \bot) \wedge ((C \wedge A) \rightarrow D) \wedge (E \rightarrow C) \wedge (A \rightarrow E) \wedge A$$

8. (Bonus exercise: 10 points)
Give a formal proof for the inference

$$\begin{array}{|l} \forall x(P \vee Q(x)) \\ \hline P \vee \forall x Q(x) \end{array}$$

Here $x$ does not occur free in $P$.
Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.